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A SUPERSYMMETRIC EFFECTIVE CHIRAL LAGRANGIAN

by

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Abstract

We construct in a manifestly supersymmetric form the leading and subleading terms in momentum for an effective supersymmetric chiral Lagrangian in terms of complex pions and their superpartners. A soft supersymmetry breaking term is included and below the supersymmetry breaking scale the Lagrangian reduces to the usual bosonic chiral Lagrangian in terms of real pions.

Supersymmetry is an attractive candidate for constructing models beyond the Standard Model [1] mainly because of the cancellation of logarithmic divergences in higher order corrections thereby providing a solution to the naturalness nature of the hierarchy problem which inevitably occurs when attempts are made to construct models in which new physics is postulated between currently accessible energies and the Planck scale. However such supersymmetric extensions of the Standard Model still require the existence of fundamental Higgs particles, albeit in chiral supermultiplets. There has to date been no evidence of the existence of fundamental scalars and one is naturally led to speculate that such Higgs particles are not fundamental particles, but are low energy manifestations of the effects of some new physics at a higher scale, which forms condensates when the couplings associated with that new physics become strong.

It is interesting to consider the possibility that such new physics exists in conjunction with supersymmetry and that furthermore the condensation scale of this new physics is large compared with the supersymmetry breaking scale. It has been pointed out [2] that the conventional fermion condensate of standard technicolour theories cannot occur without at the same time breaking supersymmetry. Such a breaking would not be consistent with the scenario proposed here since the breaking scales would then necessarily be the same. However as pointed out in ref.[2], if it were not the fermions but a composite formed out of the scalar supermultiplets of the fermions that acquired a vacuum expectation value, then it is indeed possible to break the internal chiral symmetry, whilst at the same time preserving the supersymmetry. If that were actually the case then there would be an important energy regime between the supersymmetry breaking scale and the chiral symmetry breaking scale where physical processes could be described in terms of a supersymmetric extension of the effective chiral Lagrangian proposed by Gasser and Leutwyler [3]. In such a model the pions would be replaced by chiral supermultiplets with each pion having an associated Majorana fermion (the "piino"). The effective action as in the case of ref.[3] has an infinite number of terms with arbitrary coefficients, but can be systematically expanded in powers of momentum (scaled by the chiral-symmetry breaking scale). Internal loops of the "piinos" would have a significant effect on calculations which have been hitherto performed using the chiral Lagrangian technique and there would be particularly interesting consequences for the restoration of unitarity as more terms in the effective Lagrangian are taken into consideration.

The requirement that the effective Lagrangian must be invariant under (N=1) supersymmetry transformations order by order in momentum severely restricts the terms that it may contain. As a method of obtaining the leading term, we begin by considering a super-

symmetric Higgs model. It is important to note that such a model differs in a crucial way from the effective chiral Lagrangian model proposed in that it contains fundamental Higgs fields other than the Goldstone bosons and their superpartners. We use this model for two purposes. The first is as a demonstration of how a model with a linearly realised chiral symmetry and supersymmetry can be broken spontaneously into a model in which only the vector symmetry is linearly realised, and the axial symmetry is realised by Goldstone bosons, which transform in a non-linear manner and at the same time preserve the supersymmetry. The second is as an aid to construct the leading momentum term of the effective theory.

The most general supersymmetric action of n chiral superfields, Φ_i, may be written as

$$I = \int d^8 z \overline{\Phi}_i \Phi_i + \int d^6 s W(\Phi) + \int d^6 \overline{s} \overline{W}(\overline{\Phi}) \tag{1}$$

where the superpotential $W(\Phi)$ is a functional of chiral superfields alone. Because we wish to construct the smallest chiral symmetry (for simplicity) we choose the manifestly $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ invariant superpotential

$$W(\Phi) = (\Sigma^2 + \Pi_a \Pi_a - f_\pi^2)\Phi \tag{2}$$

where in components we have ($y^m = x^m + i\theta\sigma^m\overline{\theta}$)

$$\begin{aligned}
\Sigma(x,\theta,\overline{\theta}) &= \sigma(y) + \sqrt{2}\theta\lambda_\Sigma(y) + \theta^2 F_\Sigma(y) \\
\Pi^a(x,\theta,\overline{\theta}) &= \pi^a(y) + \sqrt{2}\theta\lambda^a(y) + \theta^2 F^a(y) \\
\Phi(x,\theta,\overline{\theta}) &= \phi(y) + \sqrt{2}\theta\lambda_\phi(y) + \theta^2 F_\Phi(y)
\end{aligned} \tag{3}$$

where $\sigma^m = (-1, \tau^a)$, and τ^a are the 2×2 Pauli matrices, $(a = 1,2,3)$. The chiral superfields Σ and Π^a transform as a $(2,\bar{2})$ under $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ whereas the chiral superfield Φ is a singlet. Combining the Σ and Π^a fields into the matrix $H = \Sigma + i\tau.\Pi$, such that under $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ H transforms as

$$H \ \rightarrow LHR^\dagger,$$

our starting action becomes

$$I = \int d^8 z \left[\frac{1}{2} tr\left(H\overline{H}\right) + \Phi\overline{\Phi}\right] + \ \alpha \int d^6 s (detH - f_\pi^2)\Phi \ + \ \alpha \int d^6 \overline{s} (det\overline{H} - f_\pi^2)\overline{\Phi} \tag{4}$$

which has the potential

$$\begin{aligned}
V &= F_\Sigma \overline{F}_\Sigma + F^a \overline{F}^a + F_\Phi \overline{F}_\Phi \\
&= \sigma\phi\overline{\sigma}\overline{\phi} + \pi^a\phi\overline{\pi}^a\overline{\phi} + \frac{1}{4}(\sigma^2 + \pi^a\pi^a - f_\pi^2)(\overline{\sigma}^2 + \overline{\pi}^a\overline{\pi}^a - f_\pi^2)
\end{aligned} \tag{5}$$

The minimum of this potential is clearly $V = 0$ which may be achieved by giving the fields the following $SU(2)_L \otimes SU(2)_R$ symmetry breaking vacuum expectation values (VEVs) †

$$< \sigma > \; = \; f_\pi \qquad\qquad < \pi_a > \; = \; < \phi > \; = \; 0 \tag{6}$$

Importantly, as implied by the $V = 0$ minimum, no auxiliary field acquires a VEV with the above assignments so supersymmetry is manifestly not broken in this model. Using these VEVs we proceed to evaluate the fermion and boson mass matrices and arrive at the following particle spectrum: ‡

- 3 massless complex scalars, π^a
- 3 massless Majorana fermions, λ^a
- 2 massive complex scalars, σ and ϕ
- 1 massive Dirac fermion composed out of λ_Σ and λ_Φ.

where all massive particles have mass $m = 2\alpha f_\pi$.

It may be pertinent to view this spectrum in the light of the symmetries of the system. Ignoring supersymmetry for the moment, it is clear that the kinetic terms in eq.(4) have an O(8) invariance. Obviously the other two terms in this action are designed to have $SU(2)_L \otimes SU(2)_R$ invariance, and to allow invariant couplings to chiral matter fields (although this latter point is not followed up in this paper). The symmetry of the action is therefore at least chiral $SU(2)_L \otimes SU(2)_R$, but since the fields in H are complex one may ask if this can be extended. By allowing the group parameters in L and R to be considered complex, and by adjusting the transformation law to

$$H \;\rightarrow\; L^{\dagger -1} H R^{-1}$$

permits the extension to transformations under SL(2C)⊗SL(2C) and coupling to corresponding matter fields. In this scenario, the real part of the σ field mixes with the imaginary parts of the pions to form a four dimensional scalar multiplet of SL(2C) (and correspondingly the imaginary part of σ mixes with the real part of the pions to form a pseudoscalar multiplet). One chiral $SU(2)_L \otimes SU(2)_R$ multiplet contains the real part of the σ field with the real parts

† We require that f_π be taken to be real in order for the model to reduce to the usual bosonic chiral model below the supersymmetry breaking scale.

‡ The complex scalar fields are actually linear superpositions of scalar and pseudoscalar fields, such that the real parts of the massive fields transform as scalars whereas the imaginary parts transform as pseudoscalars - for the massless fields it is the other way around.

of the pions as usual, and the other contains the complementary components. The transformations under SL(2C)⊗SL(2C) make the eight component multiplet irreducible. It is immediately clear that the $detH$ remains unchanged as before. However the kinetic terms (notwithstanding their O(8) invariance) are only invariant under the chiral SU(2)⊗SU(2) subgroup of SL(2C)⊗SL(2C). It seems, therefore, that although the existence of the massless triplet of pions follows as usual from the Goldstone theorem applied to the breaking of SU(2)⊗SU(2) down to the vector subgroup, there is no corresponding broken symmetry of which the massless scalar triplet are the Goldstone bosons. Instead this is a consequence of the supersymmetry forcing the chiral superfield to be complex. Indeed we shall see shortly how soft supersymmetry breaking which still preserves the chiral SU(2)⊗SU(2) symmetry gives explicit masses to these scalars. In the absence of such soft terms one can demonstrate that the masslessness of the scalars is protected by supersymmetry with a simple calculation of the one loop correction to the scalar and pseudoscalar masses. For the pseudoscalars, which are genuine Goldstone bosons, the contributions to the mass corrections from internal boson loops and internal fermion loops vanish separately, whereas for the scalars the correction vanishes by virtue of a cancellation between internal boson loops and internal fermion loops.

Restricting ourselves to low momenta (i.e. integrating out the massive degrees of freedom) we are left with three massless chiral multiplets which contain precisely those particles which we would expect to appear in the natural supersymmetric extension of the standard bosonic chiral Lagrangian. It is important to note here that the massless and massive particles still combine into supermultiplets, so that we can consistently integrate out all the massive fields without violating the supersymmetry.

We may eliminate the massive degrees of freedom in a consistent fashion by taking the formal limit where $\alpha \to \infty$. When this is done we are left with the action

$$I = \int d^8 z \left[\Sigma\overline{\Sigma} + \Pi_a \overline{\Pi}_a + \Phi\overline{\Phi} \right] \tag{7}$$

with the superfields subject to the constraint

$$\Sigma^2 + \Pi_a \Pi_a = f_\pi^2 \tag{8}$$

This superfield equation contains three component field constraints

$$\sigma^2 + \pi_a \pi_a = f_\pi^2 \tag{9}$$

$$\sigma \lambda_\Sigma^\alpha + \pi_a \lambda_a^\alpha = 0 \tag{10}$$

$$F_\Sigma \sigma + F_a \pi_a = \frac{1}{2}(\lambda_\Sigma \lambda_\Sigma + \lambda_a \lambda_a) \tag{11}$$

The first consequence of these constraints is that the superfield Φ takes no part in the interactions - it is therefore a spectator field and will be ignored from now on. Eliminating σ, λ_Σ, and F_Σ, i.e. inserting the above constraints into the kinetic part of the Lagrangian in order to obtain the leading term in the low momentum expansion, the component field Lagrangian becomes

$$\begin{aligned}\mathcal{L} &= g_{a\bar{b}}\left[-\partial_m\pi^a\partial^m\overline{\pi}^b + \frac{i}{2}\mathcal{D}\overline{\lambda}^b.\sigma\lambda^a - \frac{i}{2}\overline{\lambda}^b\sigma.\mathcal{D}\lambda^a\right] \\ &+ \frac{1}{4}\lambda^a\lambda^b\overline{\lambda}^c\overline{\lambda}^d g_{a\bar{d},b\bar{c}} - \frac{1}{2}\lambda^c\lambda^d g_{c\bar{a},d}\overline{F}^a - \frac{1}{2}\overline{\lambda}^c\overline{\lambda}^d g_{\bar{c}a,\bar{d}}F^a + F^a\overline{F}^b g_{a\bar{b}}. \end{aligned} \tag{12}$$

The π^a ($\overline{\pi}^a$) fields are the holomorphic (antiholomorphic) coordinates on a Kähler manifold [4] whose metric, $g_{a\bar{b}}$, is given by

$$g_{a\bar{b}} = \delta^{ab} + \frac{\pi^a\overline{\pi}^b}{\sqrt{f_\pi^2 - \pi^c\pi^c}\sqrt{f_\pi^2 - \overline{\pi}^c\overline{\pi}^c}} \tag{13}$$

and $\mathcal{D}_m$ the covariant derivative

$$\mathcal{D}_m\lambda^b = \partial_m\lambda^b + \Gamma^b_{rs}(\partial_m\pi^s)\lambda^r \tag{14}$$

where Γ^b_{rs} is the connection on the Kähler manifold. The auxiliary fields are eliminated via their equations of motion

$$\overline{F}^b g_{a\bar{b}} = \frac{1}{2}\overline{\lambda}^c\overline{\lambda}^d g_{\bar{c}a,\bar{d}} \qquad\qquad F^a g_{\bar{a}b} = \frac{1}{2}\lambda^c\lambda^d g_{c\bar{a},d} \tag{15}$$

The inverse of the metric exists allowing us to write

$$\mathcal{L} = g_{a\bar{b}}\left[-\partial_m\pi^a\partial^m\overline{\pi}^b + \frac{i}{2}\mathcal{D}\overline{\lambda}^b.\sigma\lambda^a - \frac{i}{2}\overline{\lambda}^b\sigma.\mathcal{D}\lambda^a\right] + \frac{1}{4}R_{\bar{d}a\bar{c}b}\lambda^a\lambda^b\overline{\lambda}^c\overline{\lambda}^d \tag{16}$$

where $R_{\bar{d}a\bar{c}b} = g_{a\bar{d},b\bar{c}} - g^{\bar{r}m}g_{a\bar{r},b}g_{\bar{d}m,\bar{c}}$ is the Riemann curvature tensor for a Kähler manifold. This last term is precisely the quartic term in the fermion fields required to complete the supersymmetry in the supersymmetric non-linear sigma model. It is interesting to note how it occurs here, by elimination of the auxiliary field in the low momentum approximation to which we are working.

It is worthwhile pointing out at this stage that because of the normalisation of the fermions relative to that of bosons, the fermions must be considered to have associated with them a factor of $\sqrt{p}$, where p is the momentum scale. With this association all the terms in eq.(16) are of order p^2.

This Lagrangian is supersymmetric by construction and reduces to the standard bosonic chiral Lagrangian when fermionic fields are suppressed and the scalar fields are taken to be real

- it therefore represents the first order term of a momentum expansion of the supersymmetric chiral Lagrangian. Following ref.[5] we introduce the Kähler potential, V, defined by

$$g_{a\bar{b}} = \frac{\partial^2 V(\pi, \overline{\pi})}{\partial \pi^a \partial \overline{\pi}^b} \tag{17}$$

allowing our action to be re-written with the superspace Lagrangian density $V(\Pi, \overline{\Pi})$ - i.e. the bosonic Kähler potential with the complex scalars replaced by their chiral superfields. Using eq.(13) we see that the bosonic Kähler potential in this case is

$$V = \pi^c \overline{\pi}^c + \sqrt{f_\pi^2 - \pi.\pi}\sqrt{f_\pi^2 - \overline{\pi}.\overline{\pi}} \tag{18}$$

We can make contact with the usual bosonic chiral Lagrangian by performing a change of variables

$$\Pi^a \to \Pi^a = \Pi'^a \frac{f_\pi}{\sqrt{\Pi'.\Pi'}} \sin\left(\frac{\sqrt{\Pi'.\Pi'}}{f_\pi}\right) \tag{19}$$

The action may now be written

$$I = \frac{f_\pi^2}{2} \int d^8 z tr G G^\dagger \tag{20}$$

where we have introduced the matrix (dropping the primes)

$$G = \exp(i\tau.\Pi / f_\pi) \tag{21}$$

to display explicitly the previous statement that the pions are coordinates of the manifold of SL(2C). We therefore consider the matrix valued chiral superfield

$$G(x, \theta, \overline{\theta}) = \exp\left(\frac{i}{f_\pi}\tau.\Pi\right) = g(y) + \sqrt{2}\theta\psi(y) + \theta^2 F_G(y) \tag{22}$$

with component fields g, ψ and F_G given by

$$g(x) = G(x, \theta, \overline{\theta})|, \quad \psi_\alpha(x) = \frac{1}{\sqrt{2}} D_\alpha G(x, \theta, \overline{\theta})|, \quad F_G(x) = -\frac{1}{4} DDG(x, \theta, \overline{\theta})| \tag{23}$$

where D_α is the supersymmetric covariant derivative and $|$ indicates that the expression is evaluated at $\theta = \overline{\theta} = 0$.

We obtain

$$g = g(\pi(x)) = \exp\left(\frac{i}{f_\pi}\tau.\pi\right) \tag{24}$$

$$\psi_\alpha = \lambda_\alpha^a \frac{\partial g}{\partial \pi^a} \tag{25}$$

and

$$F_G = -\frac{1}{2}\lambda^b\lambda^a\frac{\partial}{\partial\pi^b}\left(\frac{\partial g}{\partial\pi^a}\right) + F^a\frac{\partial g}{\partial\pi^a} \tag{26}$$

These closed expressions facilitate the geometrical interpretation of our model namely that we can relate the fermions ψ to be fermions defined in the tangent space to the Kähler manifold at the point $(\pi^a, \overline{\pi}^a)$, related to λ^a by the vielbein, $e_b^A(\pi)$, given by

$$e_b^A(\pi) \;=\; \frac{1}{2}tr\left(\tau^A\frac{\partial g}{\partial\pi^b}\right)$$

so that eq.(25) may be rewritten

$$\psi_\alpha = \delta_{AB}\tau^A e_c^B(\pi)\lambda_\alpha^c \tag{27}$$

Because supersymmetry requires that the scalar particles (the pions) be complex, contact with the bosonic chiral Lagrangian can only be made below the supersymmetry breaking scale, μ, at which the imaginary parts of the pions, π_I^a, acquire a mass. Although this scale is assumed to be far below the chiral symmetry breaking scale, f_π, it would be unnatural to include a term in the action which did not respect the initial $\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R$ symmetry. Thus we propose a soft supersymmetry breaking term

$$I_{SOFT} \;=\; -\int d^8z\frac{\mu^2}{4}\theta^2\overline{\theta}^2\left(tr\left(H\overline{H}\right) - 2f_\pi^2\right) \tag{28}$$

In terms of the pions (after substituting for the σ fields, eq.(9)) this gives a mass term for the π_I^a and a sequence of interaction terms, between the real (π_R^a) parts and the imaginary parts of the pions.

$$I_{SOFT} \;=\; \int d^4x\left(-\frac{\mu^2}{2}\pi_I^{a\;2} - \frac{\mu^2}{16f_\pi^2}\left(\pi_R^a\pi_I^a\right)^2 + ...\right) \tag{29}$$

Thus we see that the soft supersymmetry breaking term provides a mass for the imaginary part of the pion, leaving the real part (which we interpret as the usual pion) massless. The interactions will undoubtedly have an effect, but they are all suppressed by powers of μ^2/f_π^2 and thus expected to be small. We note here that the soft breaking term does not directly contribute a mass to the fermions, λ^a. However as their masslessness is no longer protected by supersymmetry, they would be expected to acquire a mass through higher order interactions. This mass is expected to be of the order of the supersymmetry breaking scale, μ, but it is quite possible that the fermion masses will be somewhat smaller than μ, and so the effect of fermion loops will persist some way below the supersymmetry breaking scale.

We now consider the extension of the model to incorporate terms higher than first order in momentum - the rule is simple; we must include all terms consistent with the $\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R$

symmetry and supersymmetry, whilst recovering the usual bosonic chiral Lagrangian in the appropriate limit. Manifestly, any term containing any power of the Kähler potential, $V = trGG^\dagger$, satisfies the first constraint. Since V is a vector superfield we immediately look at the standard kinetic term for a vector supermultiplet which by construction is of fourth power in momentum.

Introducing the matrix valued chiral superfield

$$W_\alpha = -\frac{1}{4}\overline{D}^2 D_\alpha GG^\dagger \tag{30}$$

we include in our Lagrangian the manifestly supersymmetric term

$$\frac{1}{4}D^2(trW^\alpha)(trW_\alpha) = \frac{1}{2}(trD)^2 - \frac{1}{4}trF^{lm}trF_{lm} - itr\eta.\sigma^m\partial_m tr\overline{\eta} \tag{31}$$

where in components we have

$$\begin{aligned} \eta_\alpha &= \sqrt{2}\psi_\alpha\overline{F}_G - i\sqrt{2}\sigma^m_{\alpha\dot{\alpha}}(\partial_m g)\overline{\psi}^{\dot{\alpha}} \\ \frac{1}{2}D &= F_G\overline{F}_G - \partial_m g\partial^m g + \frac{i}{2}(\partial_m\psi)\sigma^m\overline{\psi} - \frac{i}{2}\psi\sigma^m\partial_m\overline{\psi} \end{aligned} \tag{32}$$

and, as its name suggests, F_{lm} is the 4-dimensional curl of the vector field component, v_m, of the vector superfield $GG^\dagger$, where

$$v_m = \psi\sigma_m\overline{\psi} + i(g\partial_m\overline{g} - \partial_m g.\overline{g}) \tag{33}$$

Upon expansion in terms of component fields eq.(31) rapidly becomes cumbersome, but it is supersymmetric and invariant under $SU(2)_L\otimes SU(2)_R$ and it only remains for us to demonstrate that the usual bosonic chiral Lagrangian is recovered in the limit where fermionic fields are suppressed and the scalar fields are taken to be real. Recalling eqs(26 and 15) we see that F_G is quadratic in fermionic fields to leading order and so in this limit $tr\eta$ vanishes and trD is simply

$$tr\left(\partial_\mu U\partial^\mu U^\dagger\right) \tag{34}$$

where U is the unitary matrix $\exp(i\tau.\pi/f_\pi)$ with π now a real scalar field. $(TrD)^2$ therefore reduces to

$$tr\left(\partial_\mu U\partial^\mu U^\dagger\right)tr\left(\partial_\nu U\partial^\nu U^\dagger\right) \tag{35}$$

which is one of the second order terms in the usual bosonic chiral Lagrangian.

Similarly, v_m and trF_{lm} are now given by

$$\begin{aligned} v_m &= U^\dagger\partial_m U - \partial_m U^\dagger.U = 2U^{-1}\partial_m U \\ trF_{lm} &= 2tr\left[U^{-1}\partial_m U.U^{-1}\partial_l U - U^{-1}\partial_l U.U^{-1}\partial_m U\right] \end{aligned} \tag{36}$$

where we have exploited the fact that U is a unitary matrix. TrF_{lm} vanishes (as expected from the fact that v_m is a pure gauge) and so eq.(31) only contributes one term to the higher order parts of the bosonic chiral Lagrangian.

Returning to eq.(30) we are able to form a second, independent, SU(2)$_L\otimes$SU(2)$_R$ invariant

$$\frac{1}{4}D^2 tr\left(W^\alpha W_\alpha\right) \tag{37}$$

This time we need to consider trD^2 and $trF^{lm}F_{lm}$. TrD^2 gives us nothing new, but, whereas trF_{lm} vanishes in the bosonic limit, $trF^{lm}F_{lm}$ does not and we recover the term

$$tr\left(\partial_l U^\dagger \partial_m U\right) tr\left(\partial^l U^\dagger \partial^m U\right) \tag{38}$$

This is the second and final addition to the first order term required to duplicate the bosonic chiral Lagrangian. From this point of view therefore, the full supersymmetric Lagrangian to second order can be written

$$\mathcal{L} \;=\; D^2\overline{D}^2 trGG^\dagger \;+\; \alpha D^2(trW^\rho)(trW_\rho) \;+\; \beta D^2 tr(W^\rho W_\rho) \tag{39}$$

where α and β are arbitrary coefficients corresponding to the two arbitrary coefficients in the higher order terms of the bosonic chiral Lagrangian.

So far we have been considering terms which are by construction invariant under the supergauge transformation

$$V \rightarrow V + \Phi + \overline{\Phi} \tag{40}$$

where Φ is an arbitrary chiral superfield. There is no a priori reason to impose this constraint and we now go on to consider terms which do not obey this symmetry (whereas these terms are not expected to give us anything new in the bosonic limit, they are allowed by supersymmetry and will be necessary for higher order supersymmetry effects).

We introduce the manifestly chiral (matrix valued) superfield

$$Z \;=\; -\frac{1}{4}\overline{D}^2 GG^\dagger \;=\; Z_\phi + \sqrt{2}\theta Z_\lambda + \theta^2 Z_F \tag{41}$$

with component fields

$$\begin{aligned} Z_\phi &= g\overline{F}_G \\ Z_{\rho\lambda} &= \psi_\rho \overline{F}_G + ig\sigma^m_{\rho\dot\alpha}\partial_m\overline{\psi}^{\dot\alpha} \\ Z_F &= F_G\overline{F}_G + g\Box\overline{g} - i\psi\sigma^m\partial_m\overline{\psi} \end{aligned} \tag{42}$$

where we note that Z_F is the only component field containing a term which does not depend explicitly on the fermions.

The first order term involving Z,

$$D^2 trZ = D^2\overline{D}^2 trGG^\dagger \tag{43}$$

simply repeats eq.(20). Proceeding, we form the next order manifestly supersymmetric term

$$D^2(ZZ) = 2Z_\phi Z_F - Z_\lambda Z_\lambda \tag{44}$$

noting that each term in this expression is at least quadratic in fermions. Most importantly therefore, this term vanishes in the bosonic limit.

A typical term in eq.(44) (from $Z_\lambda Z_\lambda$) is

$$\psi\psi\overline{F}_G\overline{F}_G \tag{45}$$

and using our earlier observations that F_G is quadratic in fermions to leading order and that fermions have a factor $\sqrt{p}$ associated with them we see that this term is of order p^3. We therefore have a completely novel feature belonging to the supersymmetric form of the chiral Lagrangian, namely the next term in the momentum expansion is of order p^3 - filling the gap in the usual bosonic model.

Proceeding to fourth order in momentum we have the additional terms

$$D^2\overline{D}^2(Z\overline{Z}) \tag{46}$$

and

$$D^2(ZZZ) = 3\left(Z_\phi^2 Z_F - Z_\lambda Z_\lambda Z_\phi\right) \tag{47}$$

where we note again that eq.(47) is at least quartic in fermionic fields and so vanishes in the bosonic limit. The term (46) only differs from the term $D^2(W^\alpha W_\alpha)$ already considered by the addition of

$$\frac{i}{4}D^2\overline{D}^2\sigma^m_{\alpha\dot{\alpha}}(D^\alpha GG^\dagger)(\overline{D}^{\dot{\alpha}}\partial_m GG^\dagger)$$

This latter expression vanishes when fermionic fields are suppressed and so the effect of (46) is also to add to the effective chiral Lagrangian new terms which are only present in the supersymmetric extension.

To obtain meaningful terms in the Lagrangian from the expressions in eqs(44, 46, 47) we need to take the trace in every possible independent way. We see that there will be two terms from eq.(44), two from eq.(46) and a further three from eq.(47). Each will have arbitrary coefficients and, where appropriate, the hermitian conjuate term is to be added, again with an arbitrary coefficient.

We thus see that the number (14) of independent next to leading order terms, and hence the number of arbitrary coefficients to that order, is much larger than for the bosonic case (where there are just two). This leads to a much richer structure for the effective action, despite the fact that all but two of these terms become unimportant below the supersymmetry breaking scale, μ.

It is easy to see how this formalism can be extended to higher chiral symmetry, $\mathrm{SU(N)}_L \otimes \mathrm{SU(N)}_R$. This is simply achieved by defining the matrix H of superfields to be an $N \times N$ matrix, transforming as an $(N, \overline{N})$ of the chiral symmetry. The term $det(H)\Phi$ in the action, eq.(4), now generates higher order, nonrenormalisable terms. This does not bother us since the linear version is just taken as a guide to construct the effective chiral action. The constraint on the σ fields, eq.(9), now becomes an N^{th} order equation, from which the metric on the Kähler SL(NC) manifold can in principle be determined, although the algebra now becomes intractable.

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During the preparation of this manuscript our attention was drawn to a paper by Clark and ter Veldhuis [6], who by approaching the subject from a different viewpoint, namely by taking the limit of the minimal supersymmetric standard model as the Higgs mass becomes infinite, have arrived at a model which coincides with the leading momentum term of the model presented here.

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